Fluctuating Strings in the Universal Confining String Theory and Gluodynamics

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Abstract

The effective string theory emerging from the bilocal approximation to the Method of Vacuum Correlators in gluodynamics is shown to be well described by the 4D theory of the massive Abelian Kalb-Ramond field interacting with the string, which is known to be the low-energy limit of the Universal Confining String Theory. This correspondence follows from the agreement of the behaviour of the coefficient functions, which parametrize the gauge-invariant correlator of two gluonic field strength tensors, known from the lattice data, with their values obtained from the propagator of the Kalb-Ramond field. We discuss this correspondence in several aspects and demonstrate that the mass of the Kalb-Ramond field in this approach plays the role of the inverse correlation length of the vacuum, so that in the massless limit string picture disappears. Next, we apply the background field method, known in the theory of nonlinear sigma models, to obtain the action, which is quadratic in quantum fluctuations around a given (e.g. minimal) string world-sheet. Several nontrivial types of couplings of these fluctuations with the background world-sheet are obtained and discussed.

1. Introduction

Recently, a new approach to the string representation of the confining phase of gauge theories was proposed[1]. This is the so-called Universal Confining String Theory (UCST), which is the theory of an Abelian antisymmetric tensor field with a nonlinear action, interacting with the string. This field effectively substitutes an infinite number of monopoles in the 3D compact QED. It was argued in Ref. 1 that the summation over the branches of the UCST action should correspond to the summation over the string world-sheets. It was also proved in Ref. 1 that the UCST partition function, which is nothing else but the Wilson average in the 3D compact QED,

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satisfies loop equations[2] modulo contact terms. However, the statement made in Ref. 1 concerning the universality of the wave operator standing on the L.H.S. of the usual loop equations obtained from the Yang-Mills theory, which were derived and investigated in Ref. 2, was only a conjecture, and the relation of these equations to the equation of motion of the tensor field in the UCST was absolutely unclear. In order to clarify this conjecture, in Ref. 3 the loop equation for the 4D UCST partition function was derived and investigated. In particular, it was demonstrated in[3] that the wave operator of the obtained loop equation is quite different from the usual one. Also the corresponding contact term was calculated explicitly.

The low-energy limit of the UCST, in which the Wilson loop could be evaluated exactly, was discussed in Ref. 1 and investigated in the 4D case in Ref. 4, where the string tension of the Nambu-Goto term and the coupling constant of the rigidity term were calculated. The latter one occured to be negative, which means that the obtained (Euclidean) string effective action is stable[5]. Also the 4D UCST action was derived in Ref. 4 by performing the exact duality transformation, while in Ref. 1 it was done only at the semiclassical level.

However, as it was first already pointed out in Ref. 6, the non-Abelian generalization of the above described antisymmetric tensor theory is quite difficult.

Another, more phenomenological, but on the other hand adapted to the non-Abelian case approach to the problem of the string representation of the confining phase of gauge theories was developed in Ref. 7-9. It is based on the Method of Vacuum Correlators[10] (MVC) and allows one to get the information about the gluodynamics string effective action from the expansion of the Wilson average, considered as a statistical weight of this string theory, in powers of the correlation length of the vacuum. In this way, in Ref. 7 the sign of the rigidity term was found to be negative according to the present lattice data, which means that the MVC is in agreement with the dual superconductor model of confinement due to Ref. 5. In Ref. 8, the string effective action obtained in Ref. 7, was applied to the derivation of the correction to the Hamiltonian of the QCD string with quarks[11] due to the rigidity term. However, up to now it is not clear how the summation over the world-sheets could appear within this approach. The most appropriate way in this direction might lie in the accounting for the perturbative gluons, which propagate inside the Wilson loop and, as it was argued in Ref. 12, should generate world-sheet excitations. In Ref. 9, this conjecture was elaborated out by virtue of the integration over perturbative gluons in the Wilson average, which in the lowest order of perturbation theory leads to the interaction of the world-sheet elements via the exchanges of perturbative gluons, propagating in the nonperturbative background. Expanding this interaction in powers of the derivatives w.r.t. the world-sheet coordinates, we finally get in the lowest order of this so-called curvature expansion some definite correction to the rigid string coupling constant, while the string tension of the Nambu-Goto term acquires no additional contribution due to perturbative gluons and keeps its pure nonperturbative value.

In this Letter, we shall propose an alternative method of description of fluctuations of the gluodynamics string world-sheet. To this end, we shall first demonstrate in the next Section that the gluodynamics string emerging within the bilocal approximation to the MVC could be effectively described by the same action of the massive Kalb-Ramond field interacting with the string, which describes the low-energy limit of the UCST. This observation will be proved by the calculation of the coefficient functions standing at two Kronecker structures (which describe surface and boundary terms in the string effective action) in the propagator of the Kalb-Ramond field, and further comparison of them with the values of the coefficient functions standing at the corresponding structures in the gauge-invariant correlator of two gluonic field strength tensors. The latter ones are unfortunately yet not found from the gluodynamics Lagrangian, while equations

for correlators were obtained in Refs. 13 and 14 and then investigated in Ref. 15 by making use of the stochastic quantization method (see also Ref. 16, where alternative equations for correlators following from the non-Abelian Bianchi identities, which were proposed in Ref. 17 and generalized in Ref. 13, were investigated). That is why, we shall compare the coefficient functions, standing in the propagator of the Kalb-Ramond field, with the lattice data[18] concerning the behaviour of the corresponding coefficient functions in gluodynamics (see also Ref. 19, where these functions were measured in QCD with dynamical fermions). We shall see that the mass of the Kalb-Ramond field plays the role of the inverse correlation length of the vacuum, so that in the sum rules' limit, when the correlation length of the vacuum tends to infinity (see the last Ref. in 10), which in our model corresponds to the case of the massless Kalb-Ramond field, the string picture disappears, and we are left with the boundary terms only. The points described above will be the topics of the next Section.

In Section 3, we shall use the model of the gluodynamics string proposed in Section 2, to describe string world-sheet excitations. Correspondingly, for the low-energy limit of the UCST this will be an exact procedure rather than a model dependent approach. The world-sheet excitations will be described with the help of the background field method developed in Ref. 20 for nonlinear sigma models. Namely, we shall split the world-sheet coordinate into a background and quantum fluctuations, after which upon the integration over the Kalb-Ramond field we shall derive for the latter ones a quadratic action, which contains several nontrivial couplings of quantum fluctuations with the background world-sheet with- and without derivatives.

The main results of the Letter are summarized in the Conclusion.

In the Appendix, we perform rather nontrivial integration over the Kalb-Ramond field in the expression for the 4D UCST partition function.

2. A Unified Description of the Gluodynamics String and the Low-Energy Limit of the UCST

The partition function of the 4D UCST (which is nothing else but the Wilson average in the Euclidean 4D compact QED) in the low-energy limit has the form[1,4]

$$\langle W(C)\rangle = N\int DB_{\mu\nu}\exp\left[\int dx\left(-\frac{1}{12\Lambda^2}H^2_{\mu\nu\lambda}-\frac{1}{4e^2}B^2_{\mu\nu}+iB_{\mu\nu}T_{\mu\nu}\right)\right]. \tag{1}$$

Here

$$H_{\mu\nu\lambda}=\partial_\mu B_{\nu\lambda}+\partial_\nu B_{\lambda\mu}+\partial_\lambda B_{\mu\nu}$$

is a strength tensor of the field $B_{\mu\nu}$,

$$T_{\mu\nu}(x)=\int d\sigma_{\mu\nu}(x(\xi))\delta(x-x(\xi))$$

is the vorticity tensor current, e is a dimensionless coupling constant, and $\Lambda\equiv\frac{\Lambda_0}{4}\sqrt{z}$, where $z\sim \mathrm{e}^{-\frac{\mathrm{const.}}{e^2}}$, and Λ_0 is a cutoff which is necessary in 4D. Gaussian integration in (1) is carried out in the Appendix and yields the UCST low-energy action in the form

$$S_{\mathrm{UCST}}=\int d\sigma_{\lambda\nu}(x)\int d\sigma_{\mu\rho}(x')\left\langle B_{\lambda\nu}(x)B_{\mu\rho}(x')\right\rangle . \tag{2}$$

Here

$$\langle B_{\lambda\nu}(x)B_{\mu\rho}(0)\rangle \equiv \langle B_{\lambda\nu}(x)B_{\mu\rho}(0)\rangle^{(1)} + \langle B_{\lambda\nu}(x)B_{\mu\rho}(0)\rangle^{(2)}\,, \tag{3}$$

where

$$\langle B_{\lambda\nu}(x)B_{\mu\rho}(0)\rangle^{(1)} = \frac{e^2m^3}{8\pi^2}\frac{K_1\left(m\,|x|\right)}{|x|}\Big(\delta_{\lambda\mu}\delta_{\nu\rho} - \delta_{\mu\nu}\delta_{\lambda\rho}\Big), \tag{4}$$

$$\langle B_{\lambda\nu}(x)B_{\mu\rho}(0)\rangle^{(2)} = \frac{e^2m}{4\pi^2x^2}\Bigg[\Bigg[\frac{K_1\left(m\,|x|\right)}{|x|} + \frac{m}{2}\Big(K_0\left(m\,|x|\right) + K_2\left(m\,|x|\right)\Big)\Bigg]\Big(\delta_{\lambda\mu}\delta_{\nu\rho} - \delta_{\mu\nu}\delta_{\lambda\rho}\Big) +$$

$$+\frac{1}{2\,|x|}\Bigg[3\left(\frac{m^2}{4} + \frac{1}{x^2}\right)K_1\left(m\,|x|\right) + \frac{3m}{2\,|x|}\Big(K_0\left(m\,|x|\right) + K_2\left(m\,|x|\right)\Big) + \frac{m^2}{4}K_3\left(m\,|x|\right)\Bigg]\cdot$$

$$\cdot\Big(\delta_{\lambda\rho}x_\mu x_\nu + \delta_{\mu\nu}x_\lambda x_\rho - \delta_{\mu\lambda}x_\nu x_\rho - \delta_{\nu\rho}x_\mu x_\lambda\Big)\Bigg]. \tag{5}$$

In Eqs. (4) and (5), $m \equiv \frac{\Lambda}{e}$ is the mass of the Kalb-Ramond field, K_i's, $i = 0, 1, 2, 3$, stand for the Macdonald functions, and one can show that the term

$$\int d\sigma_{\lambda\nu}(x)\int d\sigma_{\mu\rho}(x')\,\langle B_{\lambda\nu}(x)B_{\mu\rho}(x')\rangle^{(2)}$$

could be rewritten as a boundary one, due to which one can immediately establish a correspondence between $\langle W(C)\rangle = \exp\left(-S_{\rm UCST}\right)$ and the Wilson average written within the bilocal approximation to the MVC. This correspondence yields the following values of the coefficient functions D and D_1, which parametrize the gauge-invariant correlator of two gluonic field strength tensors

$$D\left(m^2x^2\right) = \frac{e^2m^3}{8\pi^2}\frac{K_1(m\,|x|)}{|x|} \tag{6}$$

and

$$D_1\left(m^2x^2\right) = \frac{e^2m}{4\pi^2x^2}\left[\frac{K_1(m\,|x|)}{|x|} + \frac{m}{2}\Big(K_0(m\,|x|) + K_2(m\,|x|)\Big)\right]. \tag{7}$$

Notice, that the derivative (or curvature) expansion of the action (2), which in this case is equivalent to the $\frac{1}{m}$-expansion, was performed in Ref. 4 and up to a common constant positive factor has the form

$$S_{\rm UCST} = m^2K_0\left(\frac{\sqrt{z}}{4e}\right)\int d^2\xi\sqrt{g} - \frac{1}{4}\int d^2\xi\sqrt{g}g^{ab}\left(\partial_a t_{\mu\nu}\right)\left(\partial_b t_{\mu\nu}\right) + \frac{m}{2\pi}f\left(\frac{\sqrt{z}}{4e}\right)\int\limits_0^1 ds\sqrt{\frac{dx_\mu}{ds}\frac{dx_\mu}{ds}}. \tag{8}$$

Here g_{ab} and $t_{\mu\nu}$ stand for the induced metric and the extrinsic curvature tensor of the string world-sheet respectively,

$$f(y) \equiv \int\limits_y^{+\infty}\frac{dt}{t}K_1(t),$$

$x_\mu(s)$ in the last term on the R.H.S. of Eq. (8) parametrizes the contour C in Eq. (1), $x_\mu(0) = x_\mu(1)$, and we have omitted the full derivative term of the form $\int d^2\xi\sqrt{g}R$, where R is a scalar curvature of the world-sheet. As it was already mentioned in the Introduction, the coupling constant of the rigidity term in this expansion is negative, which means the stability of the string.

Asymptotic behaviours of the functions D and D_1 following from Eqs. (6) and (7) at $|x| \ll \frac{1}{m}$ and $|x| \gg \frac{1}{m}$ read

$$D \longrightarrow \frac{e^2m^2}{8\pi^2x^2}, \tag{9}$$

$$D_1 \longrightarrow \frac{e^2}{2\pi^2\left(x^2\right)^2} \tag{10}$$

and

$$D \longrightarrow \frac{e^2m^4}{8\sqrt{2}\pi^{\frac{3}{2}}}\frac{\mathrm{e}^{-m|x|}}{\left(m\left|x\right|\right)^{\frac{3}{2}}}, \tag{11}$$

$$D_1 \longrightarrow \frac{e^2m^4}{4\sqrt{2}\pi^{\frac{3}{2}}}\frac{\mathrm{e}^{-m|x|}}{\left(m\left|x\right|\right)^{\frac{5}{2}}} \tag{12}$$

respectively.

Let us comment on the asymptotic behaviours (9)-(12). Firstly, one can see that Eq. (10) is in agreement with the MVC, where due to Ref. 10 at the distances which are much smaller than the correlation length of the vacuum T_g,

$$D_1 \longrightarrow \frac{16\alpha_s\left(x^2\right)}{3\pi\left(x^2\right)^2}. \tag{13}$$

The difference in the numerical constants in the asymptotic behaviours (10) and (13) is due to the colour factor in the one-gluon exchange diagram, which contributes into Eq. (13). This factor, which is absent in the asymptotics (10) of the Abelian propagator, could be accounted for by the proper tuning of the charge e of the Kalb-Ramond field, if we replace the running $\alpha_s(x^2)$ in Eq. (13) by some fixed value, say approximate $\alpha_s(x^2)$ by its "frozen" value, which it acquires at the confinement scale[21] $\frac{1}{\sqrt{\sigma}}$, where σ is the string tension of the Nambu-Goto term.

However Eq. (9) agrees with MVC only in the lowest order of perturbation theory, when the perturbative part of the function D is absent, and D tends to the gluonic condensate at $|x| \to 0$, i.e. this agreement is only in a sense that $D \ll D_1$. One can see that Eq. (9) does not coincide with the behaviour of the function $D\left(x^2\right)$ of the type

$$\frac{1}{\left(x^2\right)^2}\left(\alpha\ln\left(M^2x^2\right)+\beta\right),$$

where α, β stand for some constants, and M is a certain mass parameter, which takes place in the next-to-leading order of perturbation theory[22]. This is due to the fact that such a behaviour is a specific property of non-Abelian theories[23] and presumably could not be reproduced by any Abelian model.

Secondly, comparing asymptotic behaviours (11) and (12) we see that the function D_1 falls off much faster than the function D, which is in agreement with the lattice data[18]. The exponential

falls-off of the functions D and D_1 also agree with Ref. 18, while the preexponential power-like behaviours do not. However the $\frac{1}{(x^2)^2}$-fits of these behaviours used in Refs. 18 and 19 were motivated by the short-distance asymptotics of the function D_1, which according to Eq. (10) is reproduced in our model as well.

Eqs. (11) and (12) tell us once more that the mass m of the Kalb-Ramond field in our approach should be treated as an inverse correlation length of the vacuum T_g, so that in the "string limit of QCD"[24], when the string tension of the Nambu-Goto term, σ, is kept fixed at vanishing T_g, we have a correspondence of the type $m \sim \sqrt{\frac{D(0)}{\sigma}}$. In the opposite regime when the Kalb-Ramond field is massless (or equivalently, in the strong coupling regime of the theory (1)), which corresponds to the QCD sum rules' case (see the last Ref. in 10), the "confining" part (4) of the propagator of the Kalb-Ramond field vanishes, and we are left only with the "boundary" part (5), whose contribution to $S_{\rm UCST}$ takes the form

$$\int d\sigma_{\lambda\nu}(x) \int d\sigma_{\mu\rho}(x') \left\langle B_{\lambda\nu}(x) B_{\mu\rho}(x')\right\rangle^{(2)} = \frac{e^2}{2\pi^2} \oint\limits_C dx_\mu \oint\limits_C dx'_\mu \frac{1}{(x-x')^2},$$

which could be anticipated from the very beginning. This result is also in agreement with Ref. 4, where it was shown that in the strong coupling regime of the theory (1) strings are completely suppressed when $\Lambda_0 \to +\infty$.

3. Description of the World-Sheet Excitations

In order to describe fluctuations of strings in the model (1), we shall adopt the background-field method developed in Ref. 20 for the nonlinear sigma models. The three differences of our case with the one considered in Ref. 20 are the absence of the Polyakov term in the action, flatness of the field manifold (i.e. the space-time), since we consider an arbitrary Wilson loop, not necessarily lieing on the unit sphere, and the necessity of the eventual integration over the field $B_{\mu\nu}$. Consequently, the geodesics passing through the background manifold $y_\mu(\xi)$ and the excited manifold $x_\mu(\xi) = y_\mu(\xi) + z_\mu(\xi)$, where $z_\mu(\xi)$ stands for the world-sheet fluctuation, are straight, $\rho_\mu(\xi, s) = y_\mu(\xi) + s z_\mu(\xi)$, where s denotes the arc-length parameter, $0 \le s \le 1$. The expansion of the action (2) in powers of quantum fluctuations $z_\mu(\xi)$ could be performed by virtue of the following generating functional, which is the arc-dependent term describing the interaction of the Kalb-Ramond field with the string in Eq. (1),

$$I\left[\rho(\xi,s)\right] = i \int d^2\xi B_{\mu\nu}\left[\rho(\xi,s)\right] \varepsilon^{ab} \left(\partial_a \rho_\mu(\xi,s)\right) \left(\partial_b \rho_\nu(\xi,s)\right). \tag{14}$$

Then the term containing n quantum fluctuations reads as

$$I^{(n)} = \frac{1}{n!} \frac{d^n}{ds^n} I\left[\rho(\xi,s)\right]\Bigg|_{s=0},$$

and we get from Eq. (14)

$$I^{(0)} = i \int d\sigma_{\mu\nu}(y(\xi)) B_{\mu\nu}\left[y(\xi)\right], \tag{15}$$

$$I^{(1)} = i \int d\sigma_{\mu\nu}(y(\xi)) z_\lambda(\xi) H_{\mu\nu\lambda}\left[y(\xi)\right], \tag{16}$$

and

$$I^{(2)}=i\int d^2\xi z_\nu(\xi)\varepsilon^{ab}\left(\partial_a y_\mu(\xi)\right)\Biggl(\left(\partial_b z_\lambda(\xi)\right)H_{\nu\mu\lambda}\left[y(\xi)\right]+\frac{1}{2}z_\alpha(\xi)\left(\partial_b y_\lambda(\xi)\right)\partial_\alpha H_{\nu\mu\lambda}\left[y(\xi)\right]\Biggr), \qquad (17)$$

where during the derivation of Eqs. (16) and (17) we have omitted several full derivative terms.

It is worth mentioning, that as it was discussed in Ref. 20, the terms (15)-(17) are necessary and sufficient to determine all one-loop ultraviolet divergences in the theory (14) at $s=0$. This statement is important for the model of the gluodynamics string with the action (2), proposed in the previous Section, since in the UCST case, Eq. (2) describes only its low-energy effective action, which does not suffer from the ultraviolet divergences. It is also important for the Abelian Higgs Model in the Londons' limit[25], where one can show[26] that the partition function (1) with an additional integration over metrics and $T_{\mu\nu}(x)$ corresponding to a closed surface is nothing else but a 't Hooft loop average defined on the string world-sheet.

In order to get the desirable action, quadratic in quantum fluctuations $z_\mu(\xi)$, we shall first carry out the integral

$$N\int DB_{\mu\nu}\exp\Biggl[-\int dx\left(\frac{1}{12\Lambda^2}H^2_{\mu\nu\lambda}+\frac{1}{4e^2}B^2_{\mu\nu}\right)+I^{(0)}+I^{(1)}\Biggr].$$

It occurs to be equal to

$$\exp\Biggl[-\int d\sigma_{\lambda\nu}\left(y(\xi)\right)\int d\sigma_{\mu\rho}\left(y(\xi')\right)\Biggl(\left\langle B_{\lambda\nu}\left[y(\xi)\right]B_{\mu\rho}\left[y(\xi')\right]\right\rangle^{(1)}+2z_\alpha(\xi)\cdot$$

$$\cdot\frac{\partial}{\partial y_\alpha(\xi)}\left\langle B_{\lambda\nu}\left[y(\xi)\right]B_{\mu\rho}\left[y(\xi')\right]\right\rangle^{(1)}+z_\alpha(\xi)z_\beta(\xi')\frac{\partial^2}{\partial y_\alpha(\xi)\partial y_\beta(\xi')}\left\langle B_{\lambda\nu}\left[y(\xi)\right]B_{\mu\rho}\left[y(\xi')\right]\right\rangle^{(1)}\Biggr)\Biggr], \qquad (18)$$

where from now on we shall omit all the boundary terms.

Secondly, one should substitute the saddle-point of the integral

$$\int DB_{\mu\nu}\exp\Biggl[-\int dx\left(\frac{1}{12\Lambda^2}H^2_{\mu\nu\lambda}+\frac{1}{4e^2}B^2_{\mu\nu}\right)+I^{(0)}\Biggr]$$

into Eq. (17). This saddle-point reads

$$B^{\text{extr.}}_{\mu\nu}\left[y(\xi)\right]=\frac{ie^2m^3}{2\pi^2}\int d\sigma_{\mu\nu}\left(y(\xi')\right)\frac{K_1\left(m\left|y(\xi)-y(\xi')\right|\right)}{\left|y(\xi)-y(\xi')\right|},$$

and upon its substitution into Eq. (17), accounting for Eq. (18), and making use of Eq. (4), we finally get the following value of the action quadratic in quantum fluctuations

$$S_{\text{quadr.}}=\frac{e^2m^3}{4\pi^2}\int d\sigma_{\mu\nu}\left(y(\xi)\right)\int\frac{d\sigma_{\mu\nu}\left(y(\xi')\right)}{\left|y(\xi)-y(\xi')\right|}\Biggl\{K_1-\frac{z_\alpha(\xi)(y(\xi)-y(\xi'))_\alpha}{\left|y(\xi)-y(\xi')\right|}\Biggl(\frac{2K_1}{\left|y(\xi)-y(\xi')\right|}+$$

$$+m\left(K_0+K_2\right)\Biggr)+\frac{z_\alpha(\xi)z_\beta(\xi')}{\left|y(\xi)-y(\xi')\right|}\Biggl[\delta_{\alpha\beta}\Biggl(\frac{K_1}{\left|y(\xi)-y(\xi')\right|}+$$

$$+\frac{m}{2}\left(K_0+K_2\right)\Bigg)-\frac{\left(y(\xi)-y(\xi')\right)_\alpha\left(y(\xi)-y(\xi')\right)_\beta}{|y(\xi)-y(\xi')|}\left(3\left(\frac{m^2}{4}+\frac{1}{\left(y(\xi)-y(\xi')\right)^2}\right)K_1+\right.$$

$$\left.+\frac{3m}{2\left|y(\xi)-y(\xi')\right|}\left(K_0+K_2\right)+\frac{m^2}{4}K_3\right)\Bigg]\Bigg\}-\frac{e^2m^3}{2\pi^2}\Bigg(\int d^2\xi z_\nu(\xi)h_{\nu\mu\lambda}\left[y(\xi)\right]\varepsilon^{ab}\left(\partial_a y_\mu(\xi)\right)\left(\partial_b z_\lambda(\xi)\right)+$$

$$+\frac{1}{2}\int d\sigma_{\mu\lambda}\left(y(\xi)\right)z_\nu(\xi)z_\alpha(\xi)\partial_\alpha h_{\nu\mu\lambda}\left[y(\xi)\right]\Bigg), \tag{19}$$

where

$$h_{\nu\mu\lambda}\left[y(\xi)\right]\equiv$$

$$\equiv\left[\int d\sigma_{\mu\lambda}\left(y(\xi')\right)\left(y(\xi)-y(\xi')\right)_\nu+\int d\sigma_{\nu\mu}\left(y(\xi')\right)\left(y(\xi)-y(\xi')\right)_\lambda+\int d\sigma_{\lambda\nu}\left(y(\xi')\right)\left(y(\xi)-y(\xi')\right)_\mu\right]\cdot$$

$$\cdot\frac{1}{\left(y(\xi)-y(\xi')\right)^2}\left[\frac{K_1}{|y(\xi)-y(\xi')|}+\frac{m}{2}\left(K_0+K_2\right)\right], \tag{20}$$

and everywhere in Eqs. (19) and (20) the arguments of the Macdonald functions are the same, $m\left|y(\xi)-y(\xi')\right|$. We have also kept the pure background term on the R.H.S. of Eq. (19). The other terms yield the couplings of quantum fluctuations with the background world-sheet. All of these terms except one do not contain the derivatives of quantum fluctuations.

4. Conclusion

In this Letter, we have modelled the gluodynamics string effective action by the theory of the massive Abelian Kalb-Ramond field, interacting with the string. The partition function (1) of this theory is nothing else but the low-energy expression for the partition function of the UCST, which is in fact the low-energy limit of the Wilson average in the 4D compact QED, rewritten in terms of the dual antisymmetric tensor field. This observation provides us with a unified description of the gluodynamics string and the UCST.

An approach to the gluodynamics string with the help of the theory (1) has been justified in Section 2, where it has been shown that the small- and large-distance asymptotic behaviours of the coefficient functions, which parametrize the gauge-invariant correlator of two gluonic field strength tensors within the MVC and could be extracted from the lattice measurements, are in a good agreement with the ones of the corresponding functions standing in the propagator of the Kalb-Ramond field. It has also been demonstrated that the mass of the Kalb-Ramond field plays the role of the inverse correlation length of the vacuum, so that in the massless limit strings are suppressed, which is in agreement with Refs. 4 and 24.

In Section 3, we have applied the background field formalism, developed in Ref. 20 for the nonlinear sigma models, in order to describe fluctuations of strings in our model. To this end, we have splitted the string world-sheet coordinate into a background part, corresponding to some given world-sheet (e.g. one with the minimal area), and quantum fluctuations, and using the arc-dependent string action as a generating functional, performed the expansion of the term, which

describes the interaction of the string with the Kalb-Ramond field, up to the second order in quantum fluctuations. Finally, upon the integration over the Kalb-Ramond field, we have derived an action, which is quadratic in quantum fluctuations and contains the pure background part and the terms describing the interaction of the background world-sheet with the quantum fluctuations. This action is given by formulae (19) and (20) and describes fluctuations of strings in the UCST and in our model of the gluodynamics string.

In the forthcoming paper[26], we shall demonstrate that there in fact exists a physical connection between UCST and gluodynamics based on the Abelian Higgs Model.

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Appendix. Integration over the Kalb-Ramond Field in the UCST Partition Function (1)

According to the general rule, in order to calculate the Gaussian integral (1) one should substitute its saddle-point value back into the integrand. The saddle-point equation in the momentum representation reads

$$\frac{1}{2\Lambda^2}\left(p^2 B^{\text{ext.}}_{\nu\lambda}(p)+p_\lambda p_\mu B^{\text{ext.}}_{\mu\nu}(p)+p_\mu p_\nu B^{\text{ext.}}_{\lambda\mu}(p)\right)+\frac{1}{2e^2}B^{\text{ext.}}_{\nu\lambda}(p)=iT_{\nu\lambda}(p).$$

This equation can be most easily solved by rewriting it in the following way

$$\left(p^2\mathbf{P}_{\lambda\nu,\alpha\beta}+m^2\mathbf{1}_{\lambda\nu,\alpha\beta}\right)B^{\text{ext.}}_{\alpha\beta}(p)=2i\Lambda^2T_{\lambda\nu}(p), \tag{A.1}$$

where we have introduced the following projection operators (see, for example, Appendix to the last Ref. in[15]; functional generalization of these operators has been introduced in Ref. 3)

$$\mathbf{P}_{\mu\nu,\lambda\rho}\equiv\frac{1}{2}\left(\mathcal{P}_{\mu\lambda}\mathcal{P}_{\nu\rho}-\mathcal{P}_{\mu\rho}\mathcal{P}_{\nu\lambda}\right)$$

and

$$\mathbf{1}_{\mu\nu,\lambda\rho}\equiv\frac{1}{2}\left(\delta_{\mu\lambda}\delta_{\nu\rho}-\delta_{\mu\rho}\delta_{\nu\lambda}\right),$$

where $\mathcal{P}_{\mu\nu}\equiv\delta_{\mu\nu}-\frac{p_\mu p_\nu}{p^2}$. These projection operators possess the following properties

$$\mathbf{1}_{\mu\nu,\lambda\rho}=-\mathbf{1}_{\nu\mu,\lambda\rho}=-\mathbf{1}_{\mu\nu,\rho\lambda}=\mathbf{1}_{\lambda\rho,\mu\nu}, \tag{A.2}$$

$$\mathbf{1}_{\mu\nu,\lambda\rho}\mathbf{1}_{\lambda\rho,\alpha\beta}=\mathbf{1}_{\mu\nu,\alpha\beta} \tag{A.3}$$

(the same properties hold for $\mathbf{P}_{\mu\nu,\lambda\rho}$), and

$$\mathbf{P}_{\mu\nu,\lambda\rho}\left(\mathbf{1}-\mathbf{P}\right)_{\lambda\rho,\alpha\beta}=0. \tag{A.4}$$

By virtue of properties (A.2)-(A.4), the solution of Eq. (A.1) reads

$$B^{\rm ext.}_{\lambda\nu}(p) = \frac{2i\Lambda^2}{p^2+m^2}\left[\mathbf{1}+\frac{p^2}{m^2}(\mathbf{1}-\mathbf{P})\right]_{\lambda\nu,\alpha\beta} T_{\alpha\beta}(p),$$

which, once being substituted back into partition function (1), yields for it the following expression

$$\langle W(C)\rangle = \exp\left[-\Lambda^2\int\frac{dp}{(2\pi)^4}\frac{1}{p^2+m^2}\left[\mathbf{1}+\frac{p^2}{m^2}(\mathbf{1}-\mathbf{P})\right]_{\mu\nu,\alpha\beta} T_{\mu\nu}(-p)T_{\alpha\beta}(p)\right]. \qquad (A.5)$$

Rewriting Eq. (A.5) in the coordinate representation we arrive at Eq. (2). From Eq. (A.5) one can easily see that the term on its R.H.S. proportional to the projection operator $\mathbf{1}-\mathbf{P}$ yields in the coordinate representation the boundary term, i.e. Eq. (5), whereas the term proportional to the projection operator $\mathbf{1}$ yields Eq. (4).

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